Comments on Brane World Cosmology

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In this paper we consider some constraints on brane-world cosmologies. In the first part we analyze different behaviors for the expansion of our universe by imposing constraints on the speed of sound. In the second part, we study the nature of matter on the brane world by means of the well-known energy conditions. We find that the strong energy condition must be completely violated at late stages of the universe.

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Contents

1 Introduction

Recently there has been a lot of interest in the old idea [1] that spacetime has more than four dimensions [2]. The most attractive scenario along these lines is, perhaps, the so-called "Randall Sundrum (RS) brane world" [3]. Within this framework the background metric is not flat along the extra coordinate; rather it is a slice of Anti-de Sitter (AdS) space. Of course, in this setup, everything is confined to live on the brane except for gravity itself, without conflict with observations. Generalizations of the RS model including embeddings into supergravity or string theory [4], as well as a number of interesting phenomenological issues [5], have sparked a flurry of activity and several groups have begun to search for possible experimental signatures of these kinds of models [6]. In addition, since the isometry group of the bulk continuum coincides with the conformal group of the brane, the Maldacena conjecture could be exploited [7]. Roughly speaking, the AdS/CFT correspondence may be able to set uniquely the boundary conditions for the fields on the edge. (We will briefly touch upon this issue in the next section).

On a different track, the picture of a brane world evolving in a larger spacetime gives an interesting new perspective on early universe cosmology [8, 9]. In particular, attention has been devoted to the question of the quantum creation of the world. A series of rather recent papers advance the idea that a brane bubble can nucleate spontaneously together with its AdS bulk [10, 11, 12, 13]. However, it is not clear yet whether this system could evolve towards a configuration of thermal equilibrium consistent with experimental data [14]. In this paper we elaborate on this question.

In section 2 we start by considering the very early stages of the brane world (i.e. at temperatures $T \sim 10^{27}K$). Assuming a general equation of state for the matter on the brane, we are able to constrain the evolution of the system by using standard procedures of shell stability against radial perturbations [15]. Strictly speaking, we show that tuning the speed of sound to the standard range, while (almost) consistent with a de Sitter world, may lead to unusual regimes for AdS domain walls. Bounds from the Weyl anomaly induced by the CFT that lives on the brane are also discussed. Afterwards, in section 3 we turn to much later ages of the brane world (at temperatures $T \leq 60K$). We study possible constraints from the energy conditions. In particular, we find that the matter threading the brane world is not consistent with the strong energy condition unless the present value of the Hubble constant is $H_0 \approx 30$ km s^{-1} Mpc^{-1} (recent measurements find $H_0 = 67 \pm 10$ km s^{-1} Mpc^{-1} [16]). We close in section 4 with a brief discussion.

2 The Early Brane World

2.1 The Very Early Brane World

According to the Hartle–Hawking "no boundary" proposal, the quantum state of the universe is given by an Euclidean path integral over compact metrics [17]. This picture can be adapted to the RS scenario by surgical grafting two Euclidean balls of AdS_5 [10, 11]. The evolution of the brane after creation is given by the analytical continuation of S^5 to real time, i.e., a de Sitter hyperboloid embedded in Lorentzian AdS_5 space. This brane world created from "nothing" is completely analogous to the four–dimensional de Sitter instanton, except that here the inside of the wall is filled with AdS bulk. Following [11, 12, 18] one can use the AdS/CFT correspondence to explain the behavior of the inflating instanton. In particular, CFTs generally exhibit a conformal anomaly when coupled to gravity [19]. In the above setup, this anomaly is the "carrier" of the effective cosmological constant on the brane.

Formally, the above scenario can be generalized to d-dimensional de Sitter spaces which bound $d+1$ dimensional AdS spaces. The CFT, however, is not straightforward to obtain. The classical action describing the above setup is given by,

$$S = \frac{L_p^{(3-d)}}{16\pi} \int_\Omega d^{d+1}x \sqrt{g} \left(R + \frac{d\,(d-1)}{\ell^2} \right) + \frac{L_p^{(3-d)}}{8\pi} \int_{\partial\Omega} d^d x \sqrt{\gamma}\ \mathfrak{K} + T \int_{\partial\Omega} d^d x \sqrt{\gamma}, \quad (1)$$

where $\mathfrak{K}$ stands for the trace of the extrinsic curvature of the boundary, γ is the induced metric on the brane, and T is the brane tension.[3] (The following discussion will mostly refer to $d = 4$, but most of the equations will be written for arbitrary d).

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[3]In our convention, the extrinsic curvature is defined as $\mathfrak{K}_{\Xi\Upsilon} = 1/2(\nabla_\Xi \hat{n}_\Upsilon + \nabla_\Upsilon \hat{n}_\Xi)$, where $\hat{n}^\Upsilon$ is
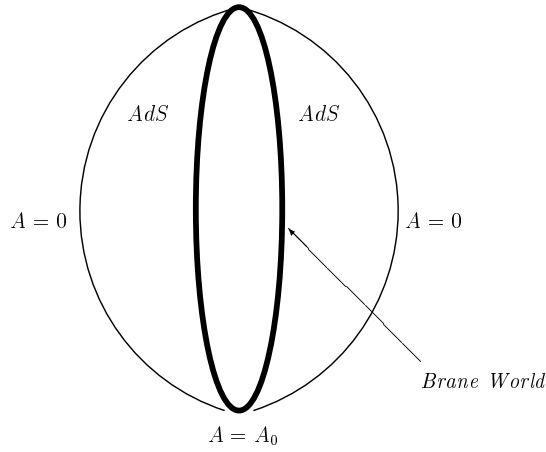
Figure 1: *De Sitter instanton. The brane is a sphere bounding to AdS Euclidean spaces. (The figure was adapted from [21]).*

The first term is the usual Einstein-Hilbert (EH) action with a negative cosmological constant ($\Lambda = -1/\ell^2$). The second term is the Gibbons-Hawking (GH) boundary term, necessary for a well-defined variational problem [20]. The third term corresponds to a constant "vacuum energy", *i.e.* a cosmological term on the boundary.

Before proceeding further, we note that the following coordinate transformations:

$$\frac{d\alpha}{d\phi} = \ell \, \tanh(\eta/\ell) \, \sin\chi \, \sin\beta \, \sin\theta \ , \quad \frac{dy}{d\eta} = \left(1 + \frac{y^2}{\ell^2}\right)^{1/2} , \tag{2}$$

give a diffeomorphism between the Euclidean AdS_5-metrics

$$ds^2 = d\eta^2 + \ell^2 \sinh^2(\eta/\ell^2)[d\chi^2 + \sin^2\chi d\Omega_3^2], \tag{3}$$

and

$$ds^2 = \left(1 + \frac{y^2}{\ell^2}\right) d\alpha^2 + \left(1 + \frac{y^2}{\ell^2}\right)^{-1} dy^2 + y^2 d\Omega_3^2. \tag{4}$$

the outward pointing normal vector to the boundary $\partial\Omega$. Capital Greek subscripts run from 0 to d and refer to the entire (d+1) dimensional spacetime, capital Latin subscripts run from 0 to $(d-1)$ and will be used to refer to the brane sub-spacetime, lower Latin subscripts run from 1 to $d-1$ and refer to constant t slices on the brane. Throughout the paper we adopt geometrodynamic units so that $G \equiv 1$, $c \equiv 1$ and $\hbar \equiv L_p^2 \equiv M_p^2$, where L_p and M_p are the Planck length and Planck mass, respectively.

4

Here $d\Omega_3^2 = d\beta^2 + \sin^2\beta(d\theta^2 + \sin^2\theta d\phi^2)$ is the volume element of the three-sphere (see Fig. 1). Thus, one can glue two of these balls along the four sphere boundaries, and then analytically continue to Lorentzian signature to obtain a four-dimensional de Sitter brane embedded in Lorentzian AdS_5 space. For simplicity we referred the above formulae to Euclidean AdS_5 balls, but it can be trivially generalized to AdS_{d+1}.

With this in mind, we consider a brane that bounds two regions of Lorentzian AdS spaces, which are conveniently described in the static chart as

$$ds^2 = -\left(k + \frac{y^2}{\ell^2}\right)dt^2 + \left(k + \frac{y^2}{\ell^2}\right)^{-1}dy^2 + y^2 d\Sigma_k^2, \tag{5}$$

where $d\Sigma_k^2$ is the corresponding metric on a $(d-1)$ dimensional space of constant curvature with metric $\bar{g}_{mn}$, and Ricci tensor $\bar{R}_{mn} = k(d-2)\bar{g}_{mn}$ with $k \in \{-1, 0, 1\}$ corresponding respectively to hyperbolic, flat and spherical geometry. The case of particular interest here is the case $k = 1$, for which the conformal anomaly of the CFT increases the effective tension of the domain wall, yielding an everlasting inflationary universe [12]. Henceforth, k shall merely be carried along as an arbitrary constant that details alternative symmetries for the brane. It should be noticed, that when the scale factor is large enough the spherical universe would be practically indistinguishable from a spatially flat universe ($k = 0$).

Applying the thin-shell formalism [22] the field equation reads,

$$T g_{\Xi\Upsilon} \delta_A^\Xi \delta_B^\Upsilon = \frac{L_p^{3-d}}{4\pi}[\mathfrak{K}_{AB} - \mathrm{tr}(\mathfrak{K}) g_{\Xi\Upsilon} \delta_A^\Xi \delta_B^\Upsilon]. \tag{6}$$

The above equation implies that the case $k = 1$ corresponds to an inflating universe which is forever expanding (a comprehensive analysis of a domain wall that inflates, either moving through the bulk or with the bulk inflating too, was first discussed by Chamblin–Reall [8]).

2.2 Matter–driven Expansion

Inflation will only be useful if it comes to an end. In the spirit of [23] we will assume that the world is created with the matter fields in their ground state, and when it starts falling under the action of the higher dimensional space the matter fields become excited. If this is the case, the subsequent expansion results from matter on the brane. We take the energy momentum tensor to be

$$\tilde{T}_{AB} = -T_c \gamma_{AB} + \tilde{\rho}\, u_A u_B + \tilde{p}\,(\gamma_{AB} + u_A u_B), \tag{7}$$

corresponding to matter with energy density $\tilde{\rho}$ and pressure $\tilde{p}$. Here, u_A stands for the velocity of a piece of stress-energy in the co–moving system ($u_A u^A = -1$), tuning the

vacuum energy to be $T_c = (d-1)/(4\pi\ell L_p^{d-3})$. From now on, to simplify notation, we denote by $\tilde{T}_0^0 \equiv -\rho(A)$, $\tilde{T}_m^m \equiv p(A)$.

The system can be decomposed into falling shells (which do not interact with each other or with the environment that generates the metric), with trajectories described by the scale factor $A(\tau)$. In other words, while the brane-world is sweeping through the $(d+1)$ dimensional bulk, the change in the internal energy is compensated by the work done by the internal forces,

$$\frac{d}{d\tau}\rho\mathcal{S} + p\frac{d}{d\tau}\mathcal{S} = 0, \tag{8}$$

where,

$$\mathcal{S} = \frac{4\pi A^2}{(d-1)\,L_p^{3-d}}. \tag{9}$$

It is straightforward, using Eq. (6) and definitions above, to check that

$$\rho = \frac{L_p^{(3-d)}\,(d-1)}{4\pi}\frac{1}{A}\left(k + \frac{A^2}{\ell^2} + \dot{A}^2\right)^{1/2}, \tag{10}$$

and

$$p = -\frac{L_p^{(3-d)}}{4\pi}\left\{\frac{(d-2)}{A}\left(k + \frac{A^2}{\ell^2} + \dot{A}^2\right)^{1/2} + \frac{\ddot{A} + A/\ell^2}{\sqrt{k + \dot{A}^2 + A^2/\ell^2}}\right\}, \tag{11}$$

satisfy the required energy conservation (dots denote derivatives with respect to τ). It should also be noted that the jump in the second fundamental form selects the positive value of the square-root. The previous equations may be recast as

$$\dot{A}^2 = -k - A^2\left(\frac{1}{\ell^2} - \frac{16\pi^2\rho^2}{(d-1)^2\,L_p^{2(3-d)}}\right); \tag{12}$$

$$\dot{\rho} = -(d-1)\,(\rho+p)\,\frac{\dot{A}}{A}. \tag{13}$$

Now, we choose a particular equation of state, in the form $p = p(\rho)$, so as to integrate the conservation equation (13)

$$\ln(A) = -\frac{1}{(d-1)}\int\frac{d\rho}{\rho + p(\rho)}. \tag{14}$$

This relationship may be formally inverted to obtain ρ as a function of the brane "radius", $\rho = \rho(A)$. In these terms, Eq. (12) becomes

$$\dot{A}^2 = -V(A); \qquad V(A) = k + A^2\left(\frac{1}{\ell^2} - \frac{16\pi^2\rho^2}{(d-1)^2\,L_p^{2(3-d)}}\right). \tag{15}$$

This single dynamical equation completely determines the expansion of the brane.

It is important to stress that when the matter fields become excited the Hubble rate $\dot{A}/A$ has an extremum, so it is possible to expand the dynamical equation around this particular "radius" denoted by A_0. Generically we would have

$$V(A) = V(A_0) + V'(A_0)(A - A_0) + \frac{1}{2}V''(A_0)(A - A_0)^2 + \mathcal{O}((A - A_0)^3), \qquad (16)$$

where a prime denotes derivative with respect to A. To compute the various derivatives, we rewrite the conservation equation as

$$[\rho(A)A]' = -[(d - 2)\rho + (d - 1)p]. \qquad (17)$$

Differentiating once more we obtain

$$[\rho(A) A]'' = \frac{(d - 1)(\rho + p)}{A}[(d - 2) + (d - 1)v_s^2], \qquad (18)$$

where

$$v_s^2(\rho) \equiv \left.\frac{\partial p}{\partial \rho}\right|_\rho \qquad (19)$$

is the speed of sound on the brane. It is easily seen that that the first derivative of the potential

$$V'(A) = \frac{2A}{\ell^2} + \frac{32\pi^2 \rho A [(d - 2)\rho + (d - 1)p]}{(d - 1)^2 L_p^{2(3-d)}} \qquad (20)$$

vanishes if

$$\rho_0 = \frac{L_p^{(3-d)}(d - 1)}{4\pi A_0}\left(k + \frac{A_0^2}{\ell^2}\right)^{1/2}, \qquad (21)$$

and

$$p_0 = -\frac{L_p^{(3-d)}}{4\pi}\left\{\frac{(d - 2)}{A_0}\left(k + \frac{A_0^2}{\ell^2}\right)^{1/2} + \frac{A_0/\ell^2}{\sqrt{k + A_0^2/\ell^2}}\right\}. \qquad (22)$$

Furthermore,

$$V''(A) = \frac{2}{\ell^2} - \frac{32\pi^2\{[(d - 2)\rho + (d - 1)p]^2 + (d - 1)\rho(\rho + p)[(d - 2) + (d - 1)v_s^2]\}}{(d - 1)^2 L_p^{2(3-d)}} \qquad (23)$$

becomes

$$V''(A_0) = \frac{2}{\ell^2} - \frac{2A_0^2/\ell^4}{k + A_0^2/\ell^2} - \frac{2k}{A_0^2}[(d - 2) + (d - 1)v_{s0}^2]. \qquad (24)$$

The square of the expansion velocity is, at this order of approximation,

$$\dot{A}^2 = -\frac{1}{2}V''(A_0)(A - A_0)^2 + \mathcal{O}((A - A_0)^3). \qquad (25)$$

7

Thus, the equation of motion for the brane requires $V''(A_0) \leq 0$. This condition can be re-written in terms of the variable $x \equiv A_0^2/\ell^2$ as, (here for $k = -1, +1$)

$$\frac{k\,[x - (k + x)\,(d - 2)]}{(d - 1)} \leq k\,(k + x)\,v_s^2. \tag{26}$$

If we now restrict the speed of sound to lie in the standard range: $v_s \in (0, 1]$, a glance at Eq. (26) shows that if $k = -1$, then

$$(3 - d)x > (2 - d). \tag{27}$$

For $d = 2$ this implies that x should be positive, while for $d = 3$ the inequality is trivially satisfied. For $d > 3$ we get the following bound $1 < x \leq (d - 2)/(d - 3)$ – in AdS_5 this is the statement that $1 < A_0^2/\ell^2 \leq 2$. Note that we have assumed throughout that $x > 1$. Indeed, from Eq. (5) we see that if $x < 1$ the brane is localized in time and not in the bulk. It is noticeable that for $k = -1$ the brane has an effective cosmological constant which is less than zero; if $\dot{A}_0$ is a maximum, the system is not able to thermalize to a final state with no cosmological constant on the brane. Hence, there is no consistent solution minimizing the value of $\dot{A}_0$. On the other hand, if $k = 1$ (de Sitter instanton) there is no constraint on x (in this case, Eq. (26) just implies that x is positive), and the brane could develop a well–behaved cosmology. Finally, for the case $k = 0$, there is no bound from the above considerations since this is a static case.

2.3   Constraints from the Weyl Anomaly

Now we will concentrate on the case of $d = 4$ and $k = +1$. With this in mind, Eq. (12) can be re-written as

$$\dot{A}^2 = -1 - \frac{A^2}{\ell^2}\left(1 - \frac{T_{eff}^2}{T_c^2}\right), \tag{28}$$

where $T_{eff} = T_c + \tilde{\rho}$. It should be stressed that an extra term, proportional to $\tilde{\rho}^2$, appears in the r.h.s. of Eq. (28) when comparing to the standard Friedmann-Robertson-Walker cosmology (a fact already known) [8]. To match the known observations of the expanding universe, the latter has to play a negligible role at least back to the time of electron-positron annihilation and primordial nucleosynthesis. At this stage we should point out that when dealing with compactified extra dimensions, one has to stabilize the value of the radion-field (which determines the size of the extra dimension) at the beginning of nucleosynthesis so as not to get into conflict with observations [9]. Throughout this paper, however, the radion is set to the minimum of its potential. Some constraints on the equation of state for cosmology with compactified extra dimensions were recently considered in [24].

All in all, one expects that the universe evolves in a similar fashion even in the presence of branes at temperatures lower than $T \sim 10^{12} K$ (more on this below). Nevertheless, there could be a significant departure from the usual scenario at very high energy scales – i.e. at the beginning of the universe – since the expansion rate could be dominated by $\tilde{\rho}^2$. To find the possible values of $\tilde{\rho}$ (at very high energy scales) we recall that the Weyl anomaly sets the effective tension of the brane to be [11]

$$T_{eff} = \frac{3 \left(1 + A_0^2/\ell^2\right)}{4\pi L_p A_0}. \tag{29}$$

The ratio between the energy density of the matter fields and the vacuum energy at the minimum classical radius of the brane equals

$$\frac{\tilde{\rho}(A_0)}{T_c} = \frac{\ell}{A_0} \left(1 + A_0^2/\ell^2\right)^{1/2} - 1. \tag{30}$$

Expanding the square–root with the assumption $A_0 \ll \ell$ we see that[4]

$$\frac{\tilde{\rho}(A_0)}{T_c} \sim \frac{\ell}{A_0} \gg 1. \tag{31}$$

This shows that the $\tilde{\rho}^2$-term dominates the early expansion of the brane world. On the other hand, one can immediately show that the Weyl anomaly does not set any constraint on the ratio A_0/ℓ.

3   Present Epoch

3.1   Energy Conditions on the Brane

The energy conditions, encoded in the evolution of the expansion scalar governed by Raychaudhuri's equation [25], are designed to side-step, as much as possible, the need to pin down a particular equation of state. They provide simple and robust bounds on the behavior of various linear combinations of the components of the stress-energy tensor. The refinement of the energy conditions paralleled the development of powerful mathematical theorems, such us singularity theorems (guaranteeing, under certain circumstances, gravitational collapse), the proof of the zeroth law of black hole thermodynamics (the constancy of the surface gravity over the event horizon), limits on the extents to which light cones can "tip over" in strong gravitational fields (superluminal censorship), the cosmic censorship conjecture (singularities cannot be unshielded, they

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[4]Note that if one abandon the idea that the solution come with some string inspired mechanism the *AdS* radius is not constrained to be of order of Planck length.

always remain hidden by event horizons), etc. [25]. In particular, the classical singularity theorem relevant to proving the existence of the big-bang singularity relies on the strong energy condition (SEC). It is somewhat disturbing to realize that current observations seem to indicate that the SEC is violated – though weakly – somewhere between the epoch of galaxy formation and the present time, in an epoch where the cosmological temperature never exceeds 60 Kelvin [26]. It is therefore worthwhile to test whether the brane world cosmology relaxes or increases the bounds on the violation of the energy conditions. In this section we shall generalize the analysis by Visser on Friedmann–Robertson–Walker (FRW) cosmologies [26].

Let us start by setting some basic nomenclature. The weak energy condition (WEC) is the assertion that for any timelike vector ξ^A, $\tilde{T}_{AB}\xi^A\xi^B \geq 0$. The null energy condition (NEC) is satisfied if and only if, $\tilde{T}_{AB}\zeta^A\zeta^B \geq 0$ for any null vector ζ^A. The strong energy condition (SEC) holds if and only if $(\tilde{T}_{AB} - \frac{1}{2}\tilde{T}\gamma_{AB}) \geq 0$. Finally, the dominant energy condition (DEC) basically says that the locally measured energy density is always positive, and that the energy flux is timelike or null, that is $\tilde{T}_{AB}\xi^A\xi^B \geq 0$, and $\tilde{T}_{AB}\xi^A$ is not spacelike (for an introduction to this subject, see [25]). These conditions can be rephrased in terms of the energy density and the principal pressures as follows,

- WEC: $\rho > 0$, and $\forall j, \rho + p_j \geq 0$

- NEC: $\forall j, \rho + p_j \geq 0$

- SEC: $\forall j, \rho + p_j \geq 0$ and $\rho + \sum_j p_j \geq 0$

- DEC: $\rho \geq 0$, and $\forall j, p_j \in [-\rho, \rho]$.

where $j = 1, \ldots, d-1$. With these expressions in hand, one can easily verify the equivalence between FRW and braneworld cosmology with respect to WEC, NEC and DEC. One can also check that these energy conditions are not in conflict with the present data.[5] In the next section we will consider the SEC.

3.2   What is the Brane World Made of?

The most direct observational evidence for the expansion of the universe comes from the redshift of spectral lines of distance galaxies. When we look into the sky and see some object, the look-back time $\tilde{\tau}$ to that object is defined as the modulus of the difference between τ_0 (the age of the universe now) and τ the age of the universe when the light that we are receiving was emitted. If we know the velocity of expansion of the

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[5]It should be noted, that the energy conditions as defined above refer to the brane world and not to the entire spacetime. The extension of these definitions to the whole spacetime leads to NEC violation in compactified RS scenarios [27].

universe $\dot{A}$, by putting a lower bound on $\dot{A}$ we deduce an upper bound on look-back time. We warn the reader not to confuse τ_0 with the birth-time. Throughout this section we use the subscript zero to indicate present epoch. Unfortunately both usages are standard.
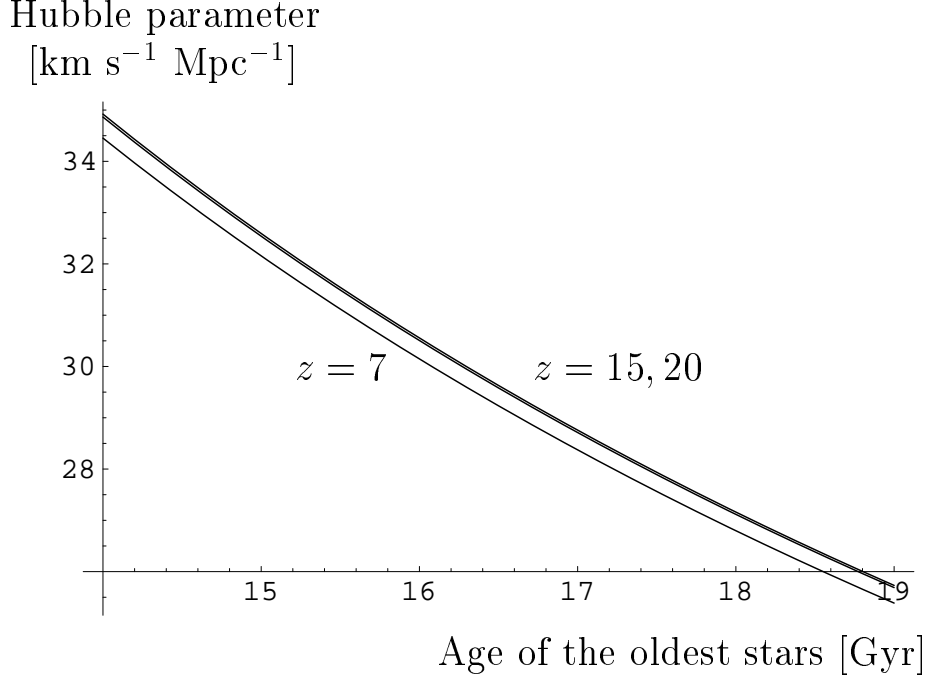

Figure 2: *Upper bounds on the Hubble parameter H_0 as a function of the age of the oldest stars for different redshifts. Note that the bounds for $z = 15, 20$ are almost identical.*

In particular, from Eqs. (10) and (11) one can trivially check that the SEC is satisfied if and only if

$$(d-3)[k + \dot{A}^2] + (d-2)\frac{A^2}{\ell^2} + \ddot{A}A \leq 0, \tag{32}$$

and

$$-[k + \dot{A}^2] + \ddot{A}A \leq 0. \tag{33}$$

Note that the condition (32) depends on d, and for AdS_3, SEC could be satisfied even with $\ddot{A} > 0$. Furthermore, contrary to the standard FRW case, SEC shows a k dependence on brane-world cosmology. We now look for violations of the SEC in

11

our brane cosmology and for that we concentrate on the condition in Eq. (32). We consider a four–dimensional flat brane ($k = 0$), which according to current experiments is the most likely to describe the world at the present epoch [16]. We further restrict the problem to $A^2 \ll \ell^2$.[6] Let us define the function $f(A) \equiv \dot{A}A$. Using Eq. (32), it is easily seen that

$$\forall A < A_0, \ f(A) \geq f(A_0). \tag{34}$$

To set some bounds on the cosmological evolution it is convenient to refer the above formulae to the redshift factor z. For a photon emitted at τ_e, z is defined as

$$z \equiv \frac{A(\tau_0)}{A(\tau_e)} - 1. \tag{35}$$

Then after integrating Eq. (34) we see that for a flat brane ($k = 0$) the SEC gives the following bound on the Hubble constant:

$$H_0 \leq \frac{2z + z^2}{2\,\tilde{\tau}(z)\,(1+z)^2}. \tag{36}$$

Without belaboring the subject, it has been known for some time now [28] that the age of the oldest stars is 16 ± 2 Gyr. The best guess for the redshift at formation of these candles is $z_f \approx 15$. Using these values in Eq. (36) yields $H_0 \approx 30$ km s^{-1} Mpc^{-1}. Even pulling z_f into $z_f = 7$ or $z_f = 20$ gives lidicous bounds, see Fig. 2. Therefore, brane-world cosmology cannot be compatible with stellar evolution and the SEC. It is easily seen from Eq. (32) that for $k = -1, +1$ the acceleration $\ddot{A}$ has to be more negative and hence that the bounds on H_0 are even more stringent than for the case discussed above.

4　Final Remarks

The fate of the universe is still uncertain [16]. Moreover, the possible existence of extra dimensions further complicate the picture. In this article, we traced a possible evolution of the brane world from the very early beginning to the formation of galaxies a few billions years later, without enforcing any particular equation of state. On the one hand, we discussed high energy scales, related to the early universe. We see that if the speed of sound is taken to lie in the standard range, a de Sitter brane world could develop a consistent cosmological scenario, whereas a similar bound may lead to unusual regimes for AdS domain walls. This constraint does not depend on the choice of equation of state. In addition, we find that if $k = 1$ the Weyl anomaly increases the effective tension on the brane in such a way that the matter density

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[6]Note that to satisfy SEC $\ddot{A}$ becomes more negative while increasing A with respect to ℓ.

$\tilde{\rho}^2$ plays a paramount role in the early universe cosmology. On the other hand, we have shown that reasonable values of the Hubble parameter imply that the strong energy condition must be violated sometime between the epoch of galaxy formation and present. Consequently, fixing the age of the universe does not just imply tuning an equation of state. To overwhelm the gravitational effects of the normal matter, we will inescapably need large quantities of matter that violates the strong energy condition, or so–called *abnormal* matter. Faced with this fact, it would perhaps be interesting to analyze whether any frozen brane-bulk interaction could improve the situation. The difficulty with this possibility will be maintaining some rather peculiar physics engendered by strong energy condition violations.

Acknowledgements

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14

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